B.Sc. 5th Semester (Honours) Examination, 2023 (CBCS)

Subject : Physics

Course : CC-XI

(Quantum Mechanics & Applications)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* of the following questions: 2×5=10

(a) The wavefunction of a particle in a stationary state with an energy E_0 at the time $t = 0$ is $\psi(x)$. After how much time will the wavefunction again be $\psi(x)$?

(b) Find the value of the constant A that makes e^{-ax^2} an eigenfunction of the operator $\left(\frac{d^2}{dx^2} - Ax^2\right)$. What is the corresponding eigenvalue? 1+1

(c) The $2p$ state for the hydrogen atom is known to be $re^{-r/2a_0}(\cos\theta)$. Find out the expectation value of r in this state.

(d) Prove that the energy eigenfunction of a free particle is doubly degenerate.

(e) Explain why normal Zeeman effect occurs only in atoms with even number of electrons.

(f) Find the value of the Lande g-factor for energy level $^3P_1$.

(g) Find the eigenfunctions of the angular momentum operator $L_z = i\hbar\frac{\partial}{\partial\phi}$.

(h) Evaluate the following commutator $\left[\vec{L}.\vec{S}, \overrightarrow{J^2}\right]$.

2. Answer *any two* of the following questions: 5×2=10

(a) Using Heisenberg's uncertainty principle, find the first Bohr radius of Hydrogen atom assuming proton to be at rest.

(b) Consider an electron impinging on a rectangular potential barrier of height $V_0 = 5\text{eV}$ and thickness $a = 10^{-10}m$. If the kinetic energy of the electron is 2·5eV, calculate the transmission coefficient.

(c) What is Zeeman effect? Describe the experimental arrangement for studying the Zeeman effect. 1+4

(d) Write down the Schrödinger equation for the electron of tritium (H_3) atom, assuming the nucleus to be stationary. Obtain the radial equation by separation of variables with special emphasis on effective potentials.

3. Answer *any two* of the following questions: 10×2=20

(a) (i) The wavefunction corresponding to the first excited state of a harmonic oscillator of frequency ω_0 is given by $\psi(x) = Ax\, e^{-\alpha x^2/2}$; $\alpha = \frac{m\omega_0}{\hbar}$. Sketch $\psi(x)$ and determine A.

(ii) Find the expectation value of the operator $\hat{x}\hat{P}_x$ in this state.

(iii) An electron of energy 342eV is confined in a one-dimensional box of length 1Å. Find out the quantum number of the electron and energy needed to excite it in the next higher level. (1+2)+4+3

(b) Write down the time-independent Schrödinger equation for the motion of the electron in a hydrogen atom, assuming that the proton is at rest.

Given: $\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$

Separate the Schrödinger equation into one radial and two angular parts. 1+3+3+3

(c) (i) A particle is confined in a one-dimensional potential will defined by

$$V_x = \begin{cases} 0, & 0 < x < a \\ \infty, & x \le 0 \text{ and } x \ge a \end{cases}$$

Obtain the energy eigenvalues and the normalized eigenfunction of the system.

(ii) Explain the 'spin-orbit coupling' of atomic electron and the consequent doubling of spectral lines with the necessary expressions. 5+5

(d) (i) Using the vector atom model, determine the possible terms corresponding to the principal quantum number $n = 3$, and compute the angle between $\vec{l}$ and $\vec{s}$ vectors for the term ${}^2D_{5/2}$.

(ii) Consider two electrons: $l_1 = 3, s_1 = \frac{1}{2}; l_2 = 1, s_2 = \frac{1}{2}$. Find the J values assuming J–J coupling. (3+3)+4

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